166

III

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No.

Part-III

MATHEMATICS, Paper - I (A)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

Note : *This question paper consists of* ***three*** *sections* ***A, B*** *and* ***C.***

SECTION - A **10×2=20**

I . *Very short answer type* questions.

(i) Attempt **all** the questions.

(ii) Each question carries **TWO** marks.

1. If $f : Q \to Q$ defined by $f(x) = 5x + 4$, then find f^{-1}.

2. Find the domain of the real function $f(x) = \sqrt{4x - x^2}$.

3. If $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{j} + 2\bar{k}$, then find a unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$.

4. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors, then find λ and μ.

5. Find the area of the parallelogram $2\bar{i} - 3\bar{j}$ and $3\bar{i} - \bar{k}$ as adjacent sides.

6. If $\sec\theta = x + \frac{1}{4x}$, prove that $\tan\theta + \sec\theta = 2x$.

7. Show that $\cos 100° \cos 40° + \sin 100° \sin 40° = \frac{1}{2}$.

8. If $\sinh x = \frac{1}{2}$, find the value of $\cosh 2x + \sinh 2x$.

9. In ΔABC, $a = 4$, $b = 5$, $c = 7$, then find the value of $\cos\left(\frac{B}{2}\right)$.

10. Find the square root of $3 + 4i$.

SECTION - B

5×4=20

II. *Short answer type* questions.

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors, then prove that the four points $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$ and $-3\bar{a} + 2\bar{b} + \bar{c}$ are coplanar.

12. Prove by vector method, the angle between the two diagonals of a Cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

13. If A is not an integral multiple of π, then prove that

$$\cos A \cdot \cos 2A \cdot \cos 4A \cdot \cos 8A = \frac{\sin 16A}{16 \sin A}.$$

14. Solve $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$.

15. Prove that $\sin^{-1}(3/5) + \sin^{-1}(8/17) = \cos^{-1}(36/85)$.

16. Prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

17. Show that $2^5 \cos^2\theta \sin^4\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$

SECTION - C

5×7=35

III. *Long answer type* questions.

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. If $f : A \to B$, $g : B \to C$ are bijections, then prove that $gof : A \to C$ is a bijection.

19. By using Mathematical induction, show that $3.5^{2n+1} + 2^{3n+1}$ is divisible by 17. $n \in N$.

20. Let $\bar{a} = i + j + k$, $\bar{b} = 2i - j + 3k$, $\bar{c} = i - j$ and $\bar{d} = 6i + 2j + 3k$. Express $\bar{d}$ in terms of $\bar{b} \times \bar{c}$, $\bar{c} \times \bar{a}$ and $\bar{a} \times \bar{b}$.

21. If A, B, C are angles of a triangle, then prove that

$$\sin^2 A/2 + \sin^2 B/2 - \sin^2 C/2 = 1 - 2\cos\frac{A}{2}\cos B/2 \sin C/2 .$$

22. If $a = 13$, $b = 14$, $c = 15$, then show that

$$R = 65/8 , \quad r = 4 , \quad r_1 = 21/2 , \quad r_2 = 12 \quad \text{and} \quad r_3 = 14.$$

23. On a tower AB of height 'h', there is a flag-staff BC at a point of d metres away from the foot of the tower. AB and BC are making equal angles. Show that the height of the flag - staff is $h\left[\frac{d^2 + h^2}{d^2 - h^2}\right]$ metres.

24. Show that one value of $\left[\frac{1 + \sin \pi/8 + i\cos \pi/8}{1 + \sin \pi/8 - i\cos \pi/8}\right]^{8/3} = -1$